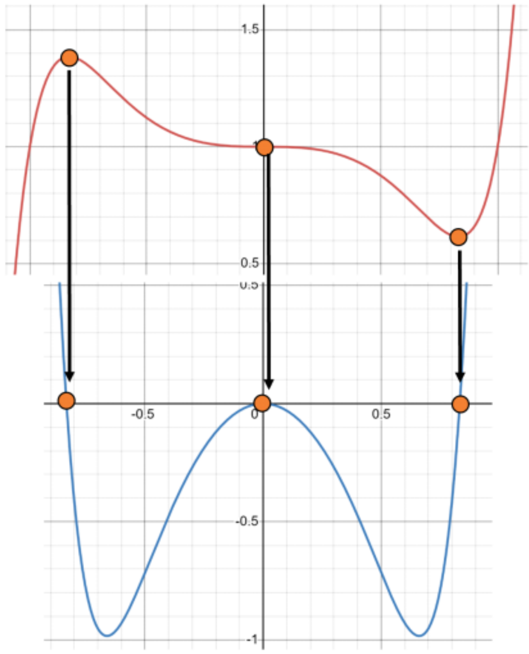
CRITICAL / STATIONARY POINTS

$f'(x) = 0$

The Math Dugout

The Double Derivative Test

We know that the graph $f'(x)$ is a measurement of the gradient of the function $f(x)$. Likewise, the graph $f''(x)$ is a measurement of the gradient of the function $f'(x)$. And so on…



- The function $f(x)$ has a critical point(s) at (a_n, b_n)
- This means that the derivative graph $f'(x)$ will have a root at $(a_n, 0)$. This is because $f'(x)$ tells us the gradient of different points on $f(x)$.

Maximum at (a_1, b_1)

- $f'(a) = 0\ (root)$
- $f''(a) = -ve$

f'' is negative as the gradient of f' will always be negative. This is because the gradient of f goes from +ve to 0 to -ve

Horizontal Point of Inflection at (a_2, b_2)

- $f'(a) = 0\ (root)$
- $f''(a) = 0$

f'' is 0 as the gradient of f' will always be 0. This is because the gradient of f goes from -ve to 0 to -ve. Or from +ve to 0 to +ve

Minimum at (a_3, b_3)

- $f'(a) = 0\ (root)$
- $f''(a) = +ve$

f'' is positive as the gradient of f' will always be positive. This is because the gradient of f goes from -ve to 0 to +ve

Example Find the nature of the critical points of function $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 6x$

Critical points when $f'(x) = 0$,

$$x^2 - 5x + 6 = 0, \qquad (x-2)(x-3) = 0 \quad, x = 2 \text{ or } x = 3$$

$f''(x) = 2x - 5$,

$$f''(2) = 2(2) - 5 = -1 = negative \therefore maximum$$

$$f''(3) = 2(3) - 5 = 1 = positive \therefore minimum$$

$f(2) = \frac{14}{3} \dots \left(2, \frac{14}{3}\right)$ *is a maximum* $\qquad f(3) = \frac{9}{2} \dots \left(3, \frac{9}{2}\right)$ *is a minimum*