THE CHAIN RULE

$$If\ y = f(u)\ and\ u = g(x)\ then\ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Consider a function, y, that is actually a combination of 2 functions.
We say that y is a function of u, $f(u)$, where in this case u is a function of x, $g(x)$.

$$y = f(u) = f(g(x))$$
$$u = g(x)$$

Scenario: Consider the function $y = (2x + 4)^5$

- We can see that $y = (u)^5, where\ u = 2x + 4$
- From this we can say that $y = f(u), where\ u = g(x)$; $f(u) = u^5, g(x) = 2x + 4$
- We can check this by substituting these back into $y = f(g(x))$ to get $y = (2x + 4)^5$

Understanding the Formula: The formula for the chain rule is as follows

$$If\ y = f(u) and\ u = g(x)\ then\ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

We say that $\frac{dy}{du} = f'(u)\ and \frac{du}{dx} = g'(x)$

Example: Find the derivative of the function $y = (2x + 4)^5$

1. Let $y = f(u)$, $u = g(x)$ such that; $f(u) = u^5$ and $g(x) = 2x + 4$
2. $\frac{dy}{du} = f'(u) = 5u^4 \quad and \quad \frac{du}{dx} = g'(x) = 2$
3. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 5u^4 \cdot 2$
   $= 10u^4$
   $= 10(2x + 4)^4 \quad Remembering\ that\ u = g(x) = 2x + 4$

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